# Research on parking strategy based on Monte Carlo simulation under annealing algorithm and particle swarm algorithm 

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#### Abstract

To address the problem of parking solutions in the parking lot, this paper establishes a mathematical model to provide a decision for the vehicles entering the parking lot, and gives the parking solution with the lowest possible parking cost based on different states. Firstly, a parking scheme evaluation model based on Monte Carlo simulation is established. The various parameters affecting the parking strategy are determined as space utilization and parking capacity. Then, a parking strategy planning model based on Monte Carlo simulation, simulated annealing algorithm, and particle swarm algorithm is developed to design the optimal parking strategy for the parking lot to minimize the average integrated cost of the population.


## 1. Introduction

When driving into a high-traffic location, choosing where to park is often difficult. Parking in a location with more empty spaces far from the end of the line can result in a longer walk; parking in a more convenient location closer to the end of the line may not have a parking space and waste more time.

Assume that there is a sufficiently sizeable one-dimensional parking lot that ends at one of the endpoints of the parking lot. It is unknown whether the parking space is empty before passing through it. There are three strategies: the first strategy is to drive into the parking lot and enter immediately when you see a parking space to ensure that there is a place, but the walking time will be longer; the second strategy is to drive to the end of the destination, if there is no parking space, and then reverse to the nearest parking space from the end. This method can find the closest parking space to the end, but the reverse will waste time; the third strategy is to enter the first time you see two side-by-side parking spaces, and if you have not seen then reverse.

We built a model to comparatively study the effect of three parking strategies and the influencing parameters. Firstly, the parking capacity and utilization rate are determined as the various parameters and set a certain variation range, and the pedestrian-vehicle speed ratio is a fixed parameter. Using the idea of Monte Carlo simulation, 1000 base-state parking lots are randomly generated. The comprehensive behavior cost is introduced to measure the effectiveness of these strategies.

Assuming that each person uses the same strategy for parking, we plan the best possible strategy and summarize the factors influencing the strategy.

## 2. The comparison of the three strategies

### 2.1 Parking condition simulation

The existing vehicle arrangement in the one-dimensional parking lot, i.e., the parking space basal distribution, is influenced by the distance of the parking space relative to the destination, the total number of parking spaces, the personal habits of car owners, and other independent factors. However, except for the distance relative to the destination, each factor does not play a key role in determining the arrangement of parking spaces. Therefore, it is considered that the probability of the base state distribution is normally distributed with the standard deviation of $\sigma$ with the distance of the parking
space relative to the destination. For the variables to take all positive values and the new probability density function to meet the normalization conditions, we transform the probability density function of the normal distribution as follows.

$$
\begin{equation*}
f(x)=\frac{2}{\sqrt{2 \pi}} e^{\frac{-x^{2}}{2 \sigma^{2}}}, x \geq 0 \tag{1}
\end{equation*}
$$

Since the parking spaces are discrete, the new probability density function should be discretized, and the variables should all be positive integers. It may be considered that the unit length of the variable taken in the probability density function image corresponds to a parking space width. In other words, the number of parking spaces apart measures the distance between the parking space and the destination, and the probability of a vehicle stopping at a parking space is the $f(x)$ integral corresponding to the unit interval taken. $i$ is the number of the corresponding parking space starting from the destination, and $P(i)$ is the probability of the $i$ th parking space, and the expression of the base state probability distribution function $P(i)$ can be obtained from the new probability density function (1).

$$
\begin{equation*}
P(i)=\frac{2}{\sqrt{2 \pi}} \int_{i-1}^{i} e^{\frac{-t^{2}}{2 \sigma^{2}}} d t, i \in N^{+} \tag{2}
\end{equation*}
$$

The corresponding function value of this function is used as the probability of each parking space being occupied to generate different parking lot basis states. Parking spaces with cars are denoted by 1 , and without cars are denoted by 0 . By varying the covariates that affect the strategy's effectiveness, a measure of the strengths and weaknesses of the strategy is derived for further analysis. Figure 1 shows the parking lot space condition when the parking space is 100 , and the space utilization rate is 60\%.


Figure 1 The parking lot space condition

### 2.2 Analysis of factors influencing parking strategy

Introducing the one-dimensional parking capacity $L$, the distance $l$ between the parking space and the destination, $v_{c}$ and $v_{w}$ are the travel speed and walking speed, respectively, and the positive and negative signs represent the presence or absence of reversal during parking, resulting in the travel consumption time $T_{c}=\frac{L \pm l}{v_{c}}$ and walking consumption time $T_{w}=l / v_{w}$. In turn, it is easy to give the stopping consumption time $T=T_{c}+T_{w}$.

From the above equation, it is reasonable to assume that the parking lot size affects the effectiveness of the strategy. And the travel speed and walking speed also affect the parking effect. The simplified
variable replaces $v_{w}$ with $v$, and the human-speed ratio is defined as $r=v_{w} / v_{c}$ to solve the actual decision-making problem better. The expression of the stopping consumption time $T$ can be rewritten as follow.

$$
\begin{equation*}
T=\frac{L \pm l}{r v}+\frac{l}{v} \tag{3}
\end{equation*}
$$

In addition, for the base-state parking space utilization, the analysis reveals that it is related to the steepness of the probability function. The steeper the function, the lower the utilization. Hence, the efficiency of the base state distribution affects the effect after parking, so it is also regarded as one of the factors affecting the strategy selection. According to $P(i)$, the utilization rate of parking spaces can be measured by the standard deviation $\sigma$. Given a growth coefficient $k$ that is constantly positive, it decreases with the increase of the base state parking space utilization rate $u$. So, the following differential expression is given.

$$
\begin{equation*}
\mathrm{d} u=k(1-u) \mathrm{d} \sigma \tag{4}
\end{equation*}
$$

The solution of $u$ is as follows, where $k$ is renamed as the base state utilization factor.

$$
\begin{equation*}
u=1-e^{-k \sigma}, k>0 \tag{5}
\end{equation*}
$$

From the discussion above, we can derive three main influencing factors that affect the parking strategy: one-dimensional parking capacity ( $L$ ), pedestrian-to-vehicle speed ratio ( $r$ ), and base-state utilization ( $u$ ). In the following, we will analyze them.

The impact of parking lot size is expressed in the indicators consumed during a certain parking process, such as energy, physical strength, and other subjective and objective indicators that change with time. The pedestrian-vehicle speed ratio affects time consumption in different ways. Based on the expression for parking time consumption ( $T$ ), it is possible to provide a comprehensive indicator to measure the goodness of different strategies in different situations. Let $w_{c}$ and $w_{w}$ represent the weight coefficients of traveling and walking, respectively, and the positive and negative signs represent the cases with and without reversing, respectively. The expression of the integrated behavioral cost $C$ is $C=\left(1-w_{c}\right) T_{c}+\left(1-w_{w}\right) T_{w}$.

Considering that the specific implementations of traveling and walking are not the same, the different weight coefficients are given to combine the behavioral costs of the two ways to be compared. Further, to better address the evaluation strategies, the human-vehicle speed ratio is set as a constant. By combining behavioral costs, different strategies in different situations can be divided to compare parallel comparisons.We use Analytic Hierarchy Process (AHP) to determine the weight coefficients of the above two behaviors. Economy, comfort, time, and convenience are taken as the evaluation criteria. After calculating and the consistency test, the weighting coefficient for traveling is 0.677 and for walking is 0.323 .

### 2.3 Measurement of strategy effectiveness

We can study the strategy selection under different vehicle base-state aggregation and dispersion states by giving different cases' base-state probability distribution functions.

Using the idea of Monte Carlo, the initial state and the process of entering the parking lot and parking the vehicle are simulated 1000 times. The capacity of the parking lot is $0-300$ parking spaces, and the utilization rate is $30 \%-100 \%$, and the total behavioral cost ( $C$ ) of the driver from entering the parking lot to walking to the destination in each simulation is recorded. After completing simulations, we average the combined behavioral cost of all simulations of a strategy. The average value is used to measure the merit of the strategy in the number of parking spaces and the utilization rate.

We have obtained three matrices corresponding to all the base state utilization ( $u$ ) of the parking lot in a certain range, all the lengths of the parking lots ( $L$ ) within a certain range, and each set of $u$ and $L$ with the corresponding integrated behavioral cost (C). We have also obtained a binary function with $u$ and $L$ as independent variables and $C$ as the dependent variable.

To visually compare the advantages and disadvantages of the schemes, scatter plots under the three strategies were produced with the independent variable as the XY axis and the dependent variable as the Z-axis, as shown in Figure 2.

Figure 3 combines the three figures in Figure 2 projected as a two-dimensional diagram, and it can be seen that there are clear boundaries between the three schemes. The yellow color in Figure 3 represents strategy 1 , the red color represents strategy 2 , and the blue color represents strategy 3. According to the boundary, the range of utilization rates and parking lot sizes that correspond optimally to each strategy are presented in Table 1. We can choose the best strategy when $u$ and $L$ are given.


Figure 2 Results of the three strategies


Figure 3 Comparison of the three strategies
Table 1 Selection of the strategy under different range

| Strategy | Range of $\boldsymbol{u}$ | Range of $\boldsymbol{L}$ |
| :---: | :---: | :---: |
| 1 | $[80 \%, 99.9 \%]$ | $[10,40]$ |
| 2 | $[50 \%, 80 \%]$ | $[40,100]$ |
| 3 | $[50 \%, 80 \%]$ | $[10,40]$ |
| 3 | $[80 \%, 99.9 \%]$ | $[40,100]$ |

## 3. The best strategy for each person

### 3.1 Calculation of the average integrated behavioral cost

Using the integrated behavioral cost formula adopted above, the integrated cost paid for the simulated multiple vehicle entry process is averaged to measure the corresponding parking strategy, thus comparing the advantages and disadvantages of different parking strategies. $k\left(1 \leq k \leq k_{m}, k \in\right.$ $N^{+}, k_{m} \geq 2$ ) is the ordinal number of different parking strategies. $S_{k}$ denotes the different parking
strategies. The average combined behavioral cost of different strategies $\overline{C_{S_{k}}}$ has the following expression.

$$
\begin{equation*}
\overline{C_{S_{k}}}=\frac{1}{k_{m}} \sum_{k=1}^{k_{m}}\left[\left(1-w_{c}\right) T_{c}+\left(1-w_{w}\right) T_{w}\right] \tag{6}
\end{equation*}
$$

### 3.2 Solving by simulated annealing algorithm

During the program's initial run using the Monte Carlo method, it was found that the program took much time to run, so it should be improved. We added a simulated annealing algorithm to find the optimal strategy solution better and faster while keeping part of the Monte Carlo procedure to reduce the range of the optimal strategy.

Similarly, the objective function in the simulated annealing algorithm is established using the average integrated behavioral cost formula.

We use the idea of the Metropolis algorithm such that $x_{k}$ as one of the solutions, where the cost $C$ can be expressed as $C=C\left(x_{k}\right), x_{k}=\left(i_{k}, j_{k}\right), k \in N^{+}$. The average composite behavioral cost difference is $\Delta C=C\left(x_{k}\right)-C\left(x_{k-1}\right)$. The expression of acceptance functions $F$ is:

$$
F=\left\{\begin{array}{c}
1, \Delta C<0  \tag{7}\\
e^{-\frac{\Delta C}{t}, \Delta C} \geq 0
\end{array}\right.
$$

We use the following recursive equation for the temperature $t$ of the recursive equation for this simulated cooling process. $\alpha$ is the recurrence factor, and $0<\alpha<1$.

Finally, we give the generation of the new solution as follows.

$$
x_{k+1}=\left\{\begin{array}{l}
\left(i_{k}+1, j_{k}\right), k \bmod 4=1  \tag{8}\\
\left(i_{k}, j_{k}+1\right), k \bmod 4=2 \\
\left(i_{k}-1, j_{k}\right), k \bmod 4=3 \\
\left(i_{k}, j_{k}-1\right), k \bmod 4=0
\end{array}\right.
$$

### 3.3 Solving with particle swarm algorithm

Same as above, set $x_{k}=\left(i_{k}, j_{k}\right), k \in N^{+}$as the position of the particle when it iterates to a certain stage. $v_{k}$ is the particle velocity, whose initial value is $v_{0}$ and the upper limit is $v_{\max } . \omega_{k}$ is the corresponding inertia factor. $a$ is the self-cognitive learning factor, and $b$ is the group cognitive learning factor. pbest $k$ is the self-cognitive optimal solution, and $g b e s t_{k}$ is the group cognitive optimal solution.

$$
\begin{equation*}
v_{k}=\omega_{k} v_{k-1}+a R\left(\text { pbest }_{k-1}-x_{k-1}\right)+b R\left(\text { gbest }_{k-1}-x_{k-1}\right), 0<\mathrm{R}<1 \tag{9}
\end{equation*}
$$

$\omega_{\text {ini }}$ and $\omega_{\text {end }}$ denote the initial inertia weights and the inertia weights corresponding to the maximum number of generations obtained by iteration, respectively. $G$ and $g$ represent the maximum number of iterations and the number of iterations, respectively.

$$
\begin{equation*}
\omega_{k}=\frac{\left(\omega_{\text {ini }}-\omega_{\text {end }}\right)(G-g)}{G-\omega_{\text {end }}}, \omega_{k}>0 \tag{10}
\end{equation*}
$$

To make the particles move in the vector space, the iterative equation is $x_{k}=x_{k-1}+v_{k-1}$.


Figure 4 The process for searching for the optimal solution
The process for searching for the optimal solution is shown in Figure 4. The parameters of the searching process in Figure 4 are settled as $L=100$ and $u=20 \%$. The parameters were chosen based on the size of a normal large mall parking lot, to make this paper a guide to parking strategies for such large parking lots.

The optimal point in Figure 4 is $(2,1,179.01)$. The optimal parking strategy is that everyone sees the second empty parking space before parking.

## References

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